8710.4600 TEACHERS OF MATHEMATICS.

Subpart 1. **Scope of practice.** A teacher of mathematics is authorized to provide to students in grades 5 through 12 instruction that is designed to develop understanding and skill in mathematical content and perspectives.

Subp. 2. **Licensure requirements.** A candidate for licensure to teach mathematics in grades 5 through 12 shall:

A. hold a baccalaureate degree from a college or university that is regionally accredited by the association for the accreditation of colleges and secondary schools;

B. demonstrate the standards for effective practice for licensing of beginning teachers in part 8710.2000; and

C. show verification of completing a Board of Teaching preparation program approved under chapter 8705 leading to the licensure of teachers of mathematics in subpart 3.

Subp. 3. **Subject matter standard.** A candidate for licensure as a teacher of mathematics must complete a preparation program under subpart 2, item C, that must include the candidate's demonstration of the knowledge and skills in items A to J.

A. A teacher of mathematics understands patterns, relations, functions, algebra, and basic concepts underlying calculus from both concrete and abstract perspectives and is able to apply this understanding to represent and solve real world problems. The teacher of mathematics must demonstrate knowledge of the following mathematical concepts and procedures and the connections among them:

(1) recognize, describe, and generalize patterns and build mathematical models to describe situations, solve problems, and make predictions;

(2) analyze the interaction between quantities and variables to model patterns of change and use appropriate representations including tables, graphs, matrices, words, ordered pairs, algebraic expressions, algebraic equations, and verbal descriptions;

(3) represent and solve problem situations that involve variable quantities and use appropriate technology;

(4) understand patterns present in number systems and apply these patterns to further investigations;

(5) apply properties of boundedness and limits to investigate problems involving sequences and series;

(6) apply concepts of derivatives to investigate problems involving rates of change;

(7) apply concepts and standard mathematical representations from differential, integral, and multivariate calculus; linear algebra, including vectors and vector spaces; and transformational operations to solve problems; and

(8) apply properties of group and field structures to mathematical investigations.

B. A teacher of mathematics understands the discrete processes from both concrete and abstract perspectives and is able to identify real world applications; the differences between the mathematics of continuous and discrete phenomena; and the relationships involved when discrete models or processes are used to investigate continuous phenomena. The teacher of mathematics must demonstrate knowledge of the following mathematical concepts and procedures and the connections among them:

(1) the application of discrete models to problem situations using appropriate representations such as sequences, vertex-edge graphs and trees, matrices, and arrays;

(2) application of systematic counting techniques to problem situations including determination of the existence of a solution, the determination of the number of possible solutions, or the optimal solution;

(3) application of discrete mathematics strategies, for example, pattern searching, organization of information, sorting, case-by-case analysis, iteration and recursion, and mathematical induction, to investigate, solve, and extend problems;

(4) exploration, development, analysis, and comparison of algorithms designed to accomplish a task or solve a problem;

(5) application of additional discrete strategies including symbolic logic and linear programming;

(6) matrices as a mathematical system and matrices and matrix operations as tools to record information and find solutions of systems of equations; and

(7) analysis of iterative and recursive algorithms to estimate the time needed in order to execute the algorithms for data likely to be encountered in problem situations.

C. A teacher of mathematics understands that number sense is the underlying structure that ties mathematics into a coherent field of study, rather than an isolated set of rules, facts, and formulae. The teacher of mathematics must demonstrate knowledge of the following mathematical concepts and procedures and the connections among them:

(1) an intuitive sense of numbers including a sense of magnitude, mental mathematics, place value, and a sense of reasonableness of results;

(2) an understanding of number systems, their properties and relations including whole numbers, integers, rational numbers, real numbers, and complex numbers;

(3) translation among equivalent forms of numbers to facilitate problem solving;

(4) application of appropriate methods of estimation of quantities and evaluation of the reasonableness of estimates;

(5) a knowledge of elementary operations, application of properties of operations, and the estimation of results;

(6) geometric and polar representation of complex numbers and the interpretation of complex solutions to equations;

(7) algebraic and transcendental numbers;

(8) numerical approximation techniques as a basis for numerical integration, numerical-based proofs, and investigation of fractals; and

(9) number theory divisibility, properties of prime and composite numbers, and the Euclidean algorithm.

D. A teacher of mathematics understands geometry and measurement from both abstract and concrete perspectives and is able to identify real world applications and to use geometric learning tools and models, including geoboards, compass and straight edge, rules and protractor, patty paper, reflection tools, spheres, and platonic solids. The teacher of mathematics must demonstrate knowledge of the following mathematical concepts and procedures and the connections among them:

(1) shapes and the ways shapes can be derived and described in terms of dimension, direction, orientation, perspective, and relationships among these properties;

(2) spatial sense and the ways shapes can be visualized, combined, subdivided, and changed to illustrate concepts, properties, and relationships;

(3) spatial reasoning and the use of geometric models to represent, visualize, and solve problems;

(4) motion and the ways in which rotation, reflection, and translation of shapes can illustrate concepts, properties, and relationships;

(5) formal and informal argument, including the processes of making assumptions; formulating, testing, and reformulating conjectures; justifying arguments based on geometric figures; and evaluating the arguments of others;

(6) plane, solid, and coordinate geometry systems including relations between coordinate and synthetic geometry, and generalizing geometric principles from a two-dimensional system to a three-dimensional system;

(7) attributes of shapes and objects that can be measured, including length, area, volume, capacity, size of angles, weight, and mass;

(8) the structure of systems of measurement, including the development and use of measurement systems and the relationships among different systems;

(9) measuring, estimating, and using measurements to describe and compare geometric phenomena;

(10) systems of geometry, including Euclidean, non-Euclidean, coordinate, transformational, and projective geometry;

(11) transformations, coordinates, and vectors, including polar and parametric equations, and the use of these in problem solving;

(12) three-dimensional geometry and its generalization to other dimensions;

(13) topology, including topological properties and transformations;

(14) extend informal argument to include more rigorous proofs; and

(15) extend work with two-dimensional right triangles including unit circle trigonometry.

E. A teacher of mathematics uses a variety of conceptual and procedural tools for collecting, organizing, and reasoning about data; applies numerical and graphical techniques for representing and summarizing data; and interprets and draws inferences from these data and makes decisions in a wide range of applied problem situations. The teacher of mathematics must demonstrate knowledge of the following mathematical concepts and procedures and the connections among them:

(1) data and its power as a way to explore questions and issues in our world;

(2) investigation through data including formulating a problem; devising a plan to collect data; and systematically collecting, recording, and organizing data;

(3) data representation to describe data distributions, central tendency, and variance through appropriate use of graphs, tables, and summary statistics;

(4) analysis and interpretation of data, including summarizing data, and making or evaluating arguments, predictions, recommendations, or decisions based on an analysis of the data; and

(5) descriptive and inferential statistics, including validity and reliability.

F. A teacher of mathematics understands how to reduce the uncertainties through predictions based on empirical or theoretical probabilities. The teacher of mathematics must demonstrate knowledge of the following mathematical concepts and procedures and the connections among them:

(1) inference, and the role of randomness and sampling in statistical claims about populations;

(2) probability as a way to describe chance or risk in simple and compound events;

(3) predicting outcomes based on exploration of probability through data collection, experiments, and simulations;

(4) predicting outcomes based on theoretical probabilities, and comparing mathematical expectations with experimental results;

(5) random variable and the application of random variable to generate and interpret probability distributions;

(6) probability theory and the link of probability theory to inferential statistics; and

(7) discrete and continuous probability distributions as a basis for making inferences about population.

G. A teacher of mathematics is able to reason mathematically, solve problems mathematically, and communicate in mathematics effectively at different levels of formality and knows the connections among mathematical concepts and procedures as well as their application to the real world. The teacher of mathematics must be able to:

(1) solve problems in mathematics by:

(a) formulating and posing problems;

(b) solving problems using different strategies, verifying and interpreting results, and generalizing the solution;

(c) using problem solving approaches to investigate and understand mathematics; and

(d) applying mathematical modeling to real world situations;

(2) reason in mathematics by:

(a) examining patterns, abstracting and generalizing based on the examination, and making convincing mathematical arguments;

(b) framing mathematical questions and conjectures, formulating counter-examples, and constructing and evaluating arguments; and

(c) using intuitive, informal exploration, and formal proof.

(3) communicate in mathematics by:

(a) expressing mathematical ideas orally, visually, and in writing;

(b) using the power of mathematical language, notation, and symbolism; and

(c) translating mathematical ideas into mathematical language, notations, and symbols; and

(4) make mathematical connections by:

(a) demonstrating the interconnectedness of the concepts and procedures of mathematics;

(b) making connections between mathematics and other disciplines;

(c) making connections between mathematics and daily living; and

(d) making connections between equivalent representations of the same concept.

H. A teacher of mathematics must:

(1) understand the historical bases of mathematics, including the contributions made by individuals and cultures, and the problems societies faced that gave rise to mathematical systems;

(2) recognize that there are multiple mathematical world views and how the teacher's own view is similar to or different from that of the students;

(3) understand the overall framework of mathematics including the:

(a) processes and consequences of expanding mathematical systems;

(b) examination of the effects of broad ideas, including operations or properties, as these ideas are applied to various systems;

(c) examination of the same object from different perspectives; and

(d) investigation of the logical reasoning that takes place within a system; and

(4) understand the role of technology, manipulatives, and models in mathematics.

I. A teacher of mathematics must demonstrate an understanding of the teaching of mathematics that integrates understanding of mathematics with the understanding of pedagogy, students, learning, classroom management, and professional development. The teacher of mathematics to preadolescent and adolescent students shall:

(1) understand and apply educational principles relevant to the physical, social, emotional, moral, and cognitive development of preadolescents and adolescents;

(2) understand and apply the research base for and the best practices of middle level and high school education;

(3) develop curriculum goals and purposes based on the central concepts of mathematics and know how to apply instructional strategies and materials for achieving student understanding of this discipline;

(4) understand the role and alignment of district, school, and department mission and goals in program planning;

(5) understand the need for and how to connect students' schooling experiences with everyday life, the workplace, and further educational opportunities;

(6) know how to involve representatives of business, industry, and community organizations as active partners in creating educational opportunities; and

(7) understand the role and purpose of cocurricular and extracurricular activities in the teaching and learning process.

J. A teacher of mathematics must understand the content and methods for teaching reading including:

(1) knowledge of reading processes and instruction including:

(a) orthographic knowledge and morphological relationships within words;

(b) the relationship between word recognition and vocabulary knowledge, fluency, and comprehension in understanding text and content materials;

(c) the importance of direct and indirect vocabulary instruction that leads to enhanced general and domain-specific word knowledge;

(d) the relationships between and among comprehension processes related to print processing abilities, motivation, reader's interest, background knowledge, cognitive abilities, knowledge of academic discourse, and print and digital text; and

(e) the development of academic language and its impact on learning and school success; and

(2) the ability to use a wide range of instructional practices, approaches, methods, and curriculum materials to support reading instruction including:

(a) the appropriate applications of a variety of instructional frameworks that are effective in meeting the needs of readers of varying proficiency levels and linguistic backgrounds in secondary settings;

(b) the ability to scaffold instruction for students who experience comprehension difficulties;

(c) selection and implementation of a wide variety of before, during, and after reading comprehension strategies that develop reading and metacognitive abilities;

(d) the ability to develop and implement effective vocabulary strategies that help students understand words including domain-specific content words;

(e) the ability to plan instruction and select strategies that help students read and understand math texts and spur student interest in more complex reading materials, including:

i. the density of ideas;

ii. concepts that build within a chapter or across chapters;

iii. use of equations to model life situations, asking students to create or restate in words or sentences the relationship between symbols and the situation being modeled;

iv. text with diagrams and graphs; and

v. use of different representations to aid students in understanding the underlying mathematical concept, matching each representation to the learning styles of different individuals; and

(f) model strategies for representing mathematical ideas in a variety of modes (literal, symbolic, graphic, and digital), which includes asking students to restate symbolic representations (numerals, equations, and graphs) in words or sentences.

Subp. 3a. **Student teaching and field experiences.** A candidate for licensure to teach mathematics must have a broad range of targeted field-based experiences, of a minimum of 100 hours prior to student teaching, that provide opportunities to apply and demonstrate competency of professional dispositions and the required skills and knowledge under this part and part 8710.2000.

Across the combination of student teaching and other field-based placements, candidates must have experiences teaching the content at three levels: kindergarten through grade 6, grades 5 through 8, and grades 9 through 12.

For initial teacher licensure, the student teaching period must be a minimum of 12 continuous weeks, full time, face-to-face, in which the candidate is supervised by a cooperating teacher, and evaluated at least twice by qualified faculty supervisors in collaboration with the cooperating teachers.

Subp. 4. **Continuing license.** A continuing license shall be issued and renewed according to the rules of the Board of Teaching governing continuing licensure.

Subp. 5. [Repealed, L 2015 c 21 art 1 s 110]

Statutory Authority: *MS s 122A.09; 122A.18*

History: *23 SR 1928; 34 SR 595; L 2015 c 21 art 1 s 110; 39 SR 822*

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